How changing physical constants and violation of local position invariance may occur?

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Light scalar fields very naturally appear in modern cosmological models, affecting such parameters of Standard Model as electromagnetic fine structure constant α, dimensionless ratios of electron or quark mass to the QCD scale , $m_{e,q}/\Lambda_{QCD}$. Cosmological variations of these scalar fields should occur because of drastic changes of matter composition in Universe: the latest such event is rather recent (redshift $z \sim 0.5$), from matter to dark energy domination. In a two-brane model (we use as a pedagogical example) these modifications are due to changing distance to "the second brane", a massive companion of "our brane". Back from extra dimensions, massive bodies (stars or galaxies) can also affect physical constants. They have large scalar charge Q_d proportional to number of particles which produces a Coulomb-like scalar field $\phi = Q_d/r$. This leads to a variation of the fundamental constants proportional to the gravitational potential, e.g. $\delta\alpha/\alpha = k_\alpha \delta(GM/rc^2)$. We compare different manifestations of this effect. The strongest limits $k_\alpha + 0.17 k_e = (-3.5 \pm 6) * 10^{-7}$ are obtained from the measurements of dependence of atomic frequencies on the distance from Sun (the distance varies due to the ellipticity of the Earth's orbit).

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I. INTRODUCTION

Changing parameters of the standard model are usually associated with the effect of massless (strictly speaking, very light) scalar fields. One candidate, much discussed in literature, is the so called *dilaton*: a very special scalar which appears in string models together with a graviton, in a massless multiplet of closed string excitations. Other scalars naturally appear in string-theory-inspired cosmological models, in which our Universe is a "brane" floating in a space of larger dimensions. The scalars are simply brane coordinates in extra dimensions.

At the other hand, available observational limits on physical constant variations at present time are quite strict, allowing only scalar coupling tiny in comparison with gravity. The only relevant scalar field recently discovered – the cosmological dark energy – also so far does not show visible variations. So one may wander why should one discuss time or space-depending physics at all?

One motivation was provided by Damour et al [1, 2] who pointed out that cosmological evolution of scalars naturally leads to their self-decoupling. Damour and Polyakov have further suggested that variations should happen when the scalars get excited by some physical change in the Universe, such as the phase transitions or other drastic change in the Equation of State (EoS) of the Universe. They considered few of them, but since the time of their paper a new fascinating EoS transition has been discovered: from matter dominated (decelerating) era to dark energy dominated (accelerating) era. It is relatively recent event, corresponding to cosmological redshift $z \approx 0.5$.

The time dependence of the disturbance related to it

can be calculated, and it turned out [4, 5] that the self-decoupling process is effective enough to explain why after this transition the variation of constants is as small as observed in laboratory experiments at the present time, as well as at Oklo (~ 2 billion years ago or $z = 0.14$) and isotopes ratios in meteorites (4.6 billion years to now, $z = 0.45 - 0$), while being at the same time consistent with possible observations [7, 8] of the variations of the electromagnetic fine structure constant at $z \sim 1$.

In this paper we discuss why and how such excitation works in some modern cosmological models at the same cosmological time of this transition. Another vast area we will address here is similar variations of constants in space, near massive bodies such as stars (Sun), pulsars, Galaxy. We will compare possible sensitivities related with different possible objects, point out limitations following from some recent experiments with atomic clocks and suggest new measurements.

Although both authors are neither cosmologists nor string theorists, we selected two-brane model as our main example, for pedagogical reasons. Variations of physical constants with distance to Sun and in cosmological time have a similar nature: in both cases it is the interaction with a massive body (Sun or heavy "second brane", respectively) via scalar fields. We hope it will be helpful to get the message across for a non-specialized reader.

II. WHICH SCALARS?

A. The dilaton and its coupling functions

Large distance interactions in Universe are dominated by gravity, which cannot be screened and get cumula-

tive effects from large bodies. Einstein's description – general relativity – uses spin-2 metric tensor $g_{\mu\nu}$ coupled to energy-momentum tensor $T^{\mu\nu} \sim \delta S_m/\delta g_{\mu\nu}$ of all matter. Thinking about large distance modification of gravity, people for a long time asked why not also add a massless (or very light) spin-0 scalar field in a similar way.

The question is what is their nature and how are they are coupled to matter. One choice (originating from literature on general relativity tests) is to work in the so called "Einstein frame" and keep gravitational part of the action in the usual form, while adding a scalar field in all other terms via a "coupling function" $A(\phi)$ in the metric

$$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}^E \tag{1}$$

where g^E is the usual "Einsteinian" metric.

$$S = \frac{1}{16\pi G} \int d^4x (g^E)^{1/2} \left[R(g^E) - 2g_{\mu\nu}^E \partial_\mu\phi\partial_\nu\phi \right] + S_{matter}(\tilde{g}_{\mu\nu}, \psi...) \tag{2}$$

If so, there is a universal source of this field ϕ,

$$\Box\phi = -4\pi G T \frac{\partial A(\phi)}{\partial\phi} \tag{3}$$

containing the trace of total matter energy-momentum tensor $T = T_\mu^\mu$. This operator T has a special meaning in physics, being a generator of the so called dilatational transformation – a change of units of all fields. That is why a field coupled to it is called a dilaton field.

Thus this field can be seen as a scale parameter: it plays the same role as e.g. a rotation angles $\vec{n}$ for the rotation operator $exp(i\vec{n}\vec{J})$ (where $\vec{J}$ is angular momentum operator), or coordinate $\vec{x}$ in displacement operator $exp(i\vec{x}\vec{P})$ (where $\vec{P}$ is momentum). So, if $\phi(x) \neq \phi(y)$ at two points x, y it means that clocks and meters *made of matter fields* (such as atoms or nuclei) are rescaled differently there. Note that this statement makes sense only because the purely geometrical Einsteinian metric g^E of space-time itself was fixed independently of $\phi(x)$, and gravitational meters - e.g. the Schwarzschield radius of a black hole – are *not* changed.

This field ϕ described so far is interacting with the total $T = \rho - 3P$ of matter (here P is the pressure). Note that it is enough to violate the equivalence principle: it is say coupled to "dust" $P = 0$ in the same way as gravity, to the mass, but not to a bag filled by black-body radiation $\rho = 3P$. Furthermore, the energy-momentum tensor trace can be expressed in terms of fundamental gauge and fermionic fields

$$T = \sum_{i=1,2,3} \frac{\beta_i(g)}{2g_i}(G_{\mu\nu}^i)^2 + \sum_f m_f \bar{\psi}_f \psi_f \tag{4}$$

Recall that Maxwellian energy-momentum tensor has zero trace and thus the gauge fields of electromagnetic, weak and strong interactions ($i = 1, 2, 3$ in the first sum) appear in it only at the quantum level, due to the so called scale anomaly induced by running coupling constant, with $\beta_i \approx -b_i g_i^2/32\pi^2$ (in the one loop order) being the corresponding beta functions. The weight of say proton or nuclei include say the electromagnetic part proportional to $\vec{E}^2 + \vec{B}^2$ while the operator above contains scalar combination $\vec{E}^2 - \vec{B}^2$ with a different coefficient, they are *not* the same.

However, even the string-theory dilaton is only simple at high string scale, where supersymmetries hold together massless supermultiplet which includes both the spin-2 graviton and spin-0 dilaton, together with a bunch of other spin $3/2,1,1/2,0$ fields. By the time one goes down to the scale at which we discuss Standard Model of particle physics these symmetries should be broken by complicated dynamical phenomena nobody understands. This may lead to a dilaton mass, and renormalize its interaction with different kind of matter differently.

In desperation, one may thus start from the opposite extreme: assume *different* coupling functions $A_i(\phi)$. in *all* term of effective low energy Lagrangian. Fortunately one can still get rid of some of them by rescaling all the matter field (we already discussed removing scalar from gravity part above). Let us think about QED or QCD action, as examples. Let us start with the fermionic kinetic/interaction term $\bar{\psi}(i\partial_\mu - A_\mu)\gamma_\nu\psi\tilde{g}_{\mu\nu}$ be modified by modifying ψ, eA_μ to absorb $A^2(\phi)$ in it: but now we have no freedom over the gauge field term

$$\frac{A_e(\phi)}{4e^2} \int d^4x \sqrt{\tilde{g}} F_{\mu\nu} F_{\mu'\nu'} \tilde{g}^{\mu\mu'} \tilde{g}^{\nu\nu'} \tag{5}$$

because we have already fixed A_μ at the previous step. The lesson is this: ϕ dependent "coupling functions" can be delegated into the gauge field coupling constants, as Bekenstein [3] did for QED (and extensively used in discussions of the variations such as [4, 5]).

A number of authors [14] (see also review[15]) suggested variations of em,weak and strong couplings should be related provided the variation comes from the high energy (right in Fig.1) end. The strong (i=3), and electroweak (i=1,2) inverse coupling constants have the following dependence on the scale μ and normalization point μ_0:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) + b_i ln(\mu/\mu_0) \tag{6}$$

In the Standard Model $2\pi b_i = 41/10, -19/6, -7$ and the couplings are related as $\alpha^{-1} = (5/3)\alpha_1^{-1} + \alpha_2^{-1}$.

Fig.1 shows two popular scenarios of Grand Unification: with the standard model as well as for its minimal supersymmetric extension (MSSM). In the latter case one can see that 3 curves cross at one point, believed to be a "root" of the three branches (electromagnetic, weak and strong). One may select the unification point for μ_0, and for example, $\mu = m_Z$ is the Z-boson mass.

(String theories lead to more complicated "trees", which however also have a singly "root", at a string scale Λ_s and bare string coupling g_s.)
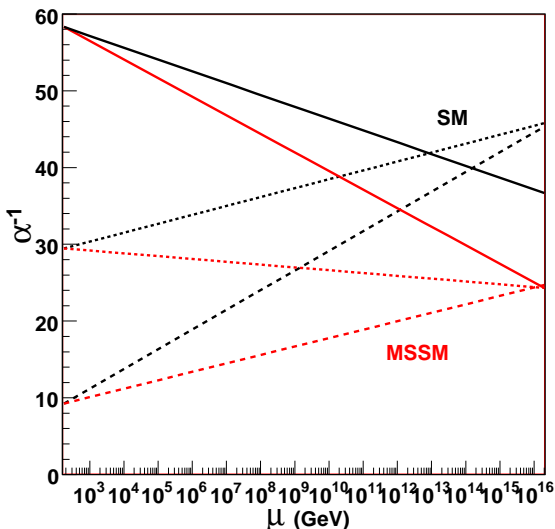
FIG. 1: From [9]. Evolution of the three gauge couplings α_1 ($SU(5)$ normalized, as the dashed line), α_2 (dotted line) and α_3 (solid line) in the SM (upper set of curves) and MSSM (lower set of curves), assuming superpartner masses are all close to the top quark mass.

So, what happens when one subject the theory to a nonzero value of the scalar field (dilaton)? A number of authors [14] (see also review[15]) suggested variations of em,weak and strong couplings should be related provided the variation comes from the high energy end. There is quite extended discussion of what may happen, but basically there are two possibilities.

If one assumes that only $\alpha_{GUT} \equiv \alpha_i(\mu_0)$ varies, the eqn (6) gives us the same shifts for all inverse couplings

$$\delta\alpha_1^{-1} = \delta\alpha_2^{-1} = \delta\alpha_3^{-1} = \delta\alpha_{GUT}^{-1} \qquad (7)$$

as the whole MSSM "tree" (Fig.1) moves up or down. If so, the variation of the strong interaction constant $\alpha_3(m_z)$ is much larger than the variation of the em constant α, $\delta\alpha_3/\alpha_3 = (\alpha_3/\alpha_1)\delta\alpha_1/\alpha_1$.

Another option is the variation of the GUT scale (μ/μ_0 in eqn (6). If so the whole "tree" at Fig.1 moves left-right and quite different relations between variations of the three coupling follows

$$\delta\alpha_1^{-1}/b_1 = \delta\alpha_2^{-1}/b_2 = \delta\alpha_3^{-1}/b_3 \qquad (8)$$

Note that now variations have different sign since the one loop coefficients b_i have different sign for 1 and 2,3.

However, it is hard to see how this relation appears dynamically, at least in cosmological models on the market. One would rather think the influence of the "coupling functions" depending on scalars in the matter terms of the Lagrangian would actually mean "curving" of the lines of the "tree" at Fig.1. Another unclear issue is the modification of lepton/quark masses, which are proportional to Higgs VEV and thus depend on the mechanism of electroweak symmetry breaking.

B. Introducing cosmological two-brane model

It would be premature at this moment to subscribe to any model, of course. However we still find some of them more dynamically natural and possibly useful, at least pedagogically, to explain our intuitive ideas in relating distinct physics together. Field and string theory provide examples of topological objects of different dimensions – branes – on which effective fermionic, gauge and scalar fields live. One of them may be our 4-d Universe with our Standard model fields, floating in a larger-dimensional space[31].

It inspired many specific cosmological models providing some dynamical realizations of Brane World Cosmology (see e.g. [12, 13] for reviews). More specifically, our pedagogical point is best served by the two-brane models, in which our 3-d brane (coordinates $\vec{x}$) is complemented by another 3-d brane, parallel to it but shifted in the 5-th dimension (we will call y). The Big Bang in this model is thus supplemented not only by the influence of the "bulk fields" living on and in between branes (gravitons and dilatons among them) but also by the second brane.

Without matter, symmetries of the model naturally balance all forces between the branes – gravity and scalar attraction is balanced by Coulomb-like vector repulsion[32]. It means that in this approximation the positions of two branes y_1, y_2 are irrelevant: branes can "levitate" in presence of each other, anywhere they chose. With extra matter added to branes this symmetry is broken and branes start their motion (making y_i a function of time t). If matter is not homogeneous, as shown in Fig.2 for the left brane, the brane bends, making it a function of 3-space point $y_i(\vec{x})$. In general, there appear 4-d scalar field $y_i(\vec{x}, t)$, describing the shape of the moving brane.

Unlike models mentioned above, nothing unusual happens with physics at high energy (right) part of Fig.1: all changes are restricted to low energy (left) end. Since gauge bosons are generated dynamically – by open strings with both ends on our brane – one can calculate how much this affect their effective Lagrangian and in principle *predict* how they are affected by the relative dilaton field, or y_1, y_2 scalars.

These two branes were originally introduced by Randall and Sundrum [6] to explain why gravity can be strong in general but exponentially weak at our brane[33] due to warping of the space-time by gravity of the companion brane.

(Another fascinating aspect of the two-brane model which we will not discuss at all is a possibility to explain the dark matter phenomenon by ascribing it to the *other* brane.)
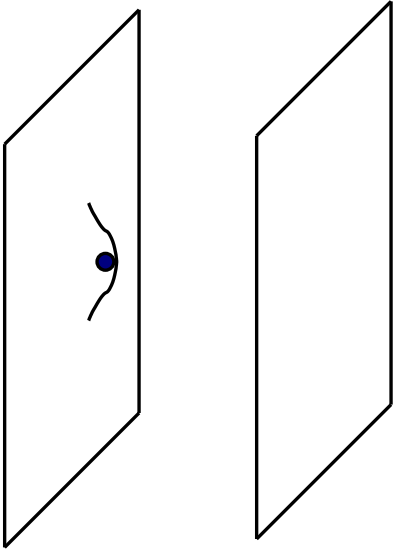
FIG. 2: Schematic picture of 2-brane construction. The co-ordinates inside two parallel branes are called $\vec{x}$ and the one normal to them y. A massive body (shaded circle) on a brane can cause its deformation, as shown, due to extra attraction to the second brane.

III. THE COSMOLOGICAL CHANGE

A. Self-decoupling mechanism

The cosmological equations follow from Einstein's equation and are well known ($q = 8\pi G$)

$$-3\ddot{a}/a = 2\dot{\phi}^2 + 8\pi G * (\rho + 3P) \qquad (9)$$

$$3(\frac{\dot{a}}{a})^2 + 3K/a^2 = 16\pi G\rho + \dot{\phi}^2 \qquad (10)$$

where $a(t)$ is the scale factor, K is related to spatial cur-vature (below we will use K=0 flat space) and ρ, P are the total energy density and pressure. The evolution of the dilaton field is given by

$$\ddot{\phi} + 3(\frac{\dot{a}}{a})\dot{\phi} = 4\pi G \sum_A A'_A(\phi)(\rho_A - 3P_A) \qquad (11)$$

where the sum in the r.h.s. is over various components of matter in Universe, with corresponding contributions to energy density and pressure[34].

In order to understand what this equation is describ-ing, let us start following the original Damour-Polyakov paper [2] and consider a single-species Universe (no sum in the r.h.s.). It is also useful to introduce "logarithmic cosmic time" $\tau = ln(a(t)) + const$ we will denote deriva-tive over it by a prime[35]. The following single eqn for $\phi(\tau)$ then follows

$$\frac{2}{3 - \phi'^2}\phi'' + (1 - P/\rho)\phi' = A'(\phi)(1 - 3P/\rho) \qquad (12)$$

which can be seen as an oscillator with exciting force (r.h.s) and the damping (the second term in the l.h.s.) containing the EoS ratio P/ρ. If its evolution can be put in as a function of time τ, this can be treated as an independent eqn.

For radiation-dominated Universe the r.h.s. ("excita-tion force") is zero ($\rho = 3P$), while l.h.s. corresponds to damped oscillator which rapidly relaxes to rest. For other single-component eras the r.h.s. is not zero, but is dominated by a single value of P/ρ (e.g. zero for matter-dominated one) and a single function $A(\phi)$: the evolu-tion is a relaxation toward the minimum of this function, which say happens at some $\phi = \phi_m$. At the minimum A' is zero and all interaction of the small-amplitude dilaton with matter vanishes.

The only time when this argument does not work is when EoS of the Universe is changing. In a simplest case, there are two comparable contributions of two types of matter. In particular, let us focus directly at the latest cosmological event of this change in which components of the energy-momentum tensor have comparable con-tributions of matter (luminous and dark) and the dark energy (cosmological constant). A single equation for ϕ (12) is no longer valid and one should solve all coupled equations (given above) together. One may find explicit solutions e.g. in [4, 5], but it is easy to guess what is happening without numerical solutions: a damped oscil-lator slides from one equilibrium position (the minimum of coupling function to matter at $\phi = \phi_m$) to a new min-imum of another coupling function to the dark energy at $\phi = \phi_d$. This slide is dominated by strong damping of cosmic oscillator, which is universal. As a result, one is able to predict the *time dependence* of the cosmological scalar field (if not the magnitude itself.). In summary: the mechanism leads to a ladder-type evolution pattern of the scalar field.

As shown in [4, 5], the last slide is such that there is ac-tually no contradiction between limits from Oklo and me-teorite isotope compositions with nonzero claim in quasar spectra [7, 8].

There is another version of scalar motion, in which people want it to slide not toward a constant value now (usually taken to be zero) but to infinity, reproducing the so called "tracking" solutions. In other words, these authors want to hit two birds with one stone and to solve the "dark energy" puzzle by the same scalar. If so, the rolling field cannot do it sufficiently quickly, and in this case the meteorite-based limits and quasar data cannot be reconciled [19].

As we will discuss in the next section, however, there is no shortage of scalar fields in modern models, and it is quite natural that different fields would be responsible for these two phenomena.
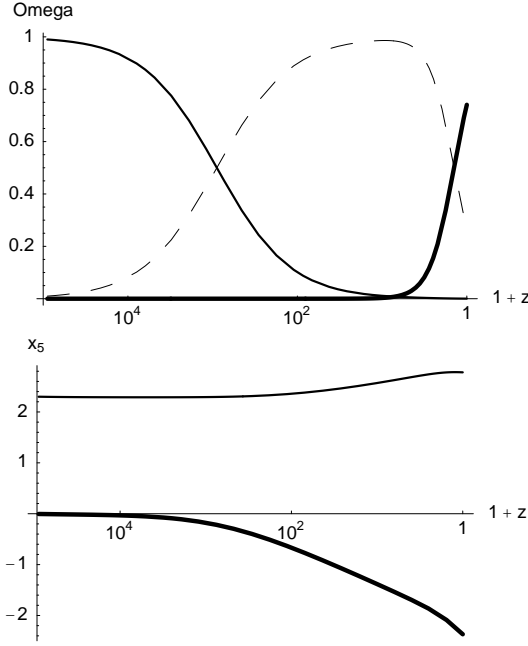
FIG. 3: From [10]. (a) Evolution of the density fraction of Universe as a function of redshift z for radiation (thin solid), matter (dashed) and the scalar fields (thick solid). At recent time the universe is accelerating because of scalar dominance. Note, that in order to explain the values for the energy density of dark energy, one has to fine–tune the parameters of the theory. For these plots $\alpha_d = 0.01$. The dark matter lives on the second brane, there is no matter on the first brane. (b) The corresponding motion of the positions of two branes. The coordinates x_5 was called y in the text.

B. Cosmic evolution of scalars in two-brane cosmology

The three scalars of the model are the bulk dilaton and 2 brane shapes $y_1(\vec{x}, t), y_2(\vec{x}, t)$ as shown in Fig. 2 : their combinations are two scalars to appear in the Lagrangian. The supersymmetric potential ensures "levitation" and defines the basic low energy Lagrangian. The parameters of the model include tensions (masses per area) of two branes and bulk cosmological constant, to be tuned as explained in [6] for static construction. To make it dynamically moving, there is additional model potential expressing the brane tension as a function of bulk dilaton scalar which is arbitrarily selected to be exponential $U \sim exp(\alpha_d\phi)$. As it turns out, the parameter α_d is restricted from above by cosmological limitations on the variations of the gravitational constant and electromagnetic fine structure constant [11]. In simple terms, it means that the two branes should be sufficiently far apart, not to cause too much disturbance in our world.

As it stands, the factors depending on scalars appear in the action in front of the Einstein-Hilbert R term. In the language of relativity practitioners it would be called "Jordan frame", which is deposited by a conformal transformation of the metric to the so called "Einstein frame" in which there are no scalars in front of the R term any more, with the cost of getting them in matter Lagrangian instead. The corresponding total low energy effective action of this model becomes of the form

$$S_{\text{EFF}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [\mathcal{R} - \frac{12\alpha_d^2}{1 + 2\alpha_d^2}(\partial\phi)^2$$

$$-\frac{6}{2\alpha_d^2 + 1}(\partial R)^2] - \int d^4x \sqrt{-g}(V_{\text{eff}}(\phi, R) + W_{\text{eff}}(\phi, R))$$

$$+ S_m^{(1)}(\Psi_1, A^2(\phi, R)g_{\mu\nu}) + S_m^{(2)}(\Psi_2, B^2(\phi, R)g_{\mu\nu}) \quad (13)$$

where one finds two tensions (or cosmological constants) of the two branes $V_{\text{eff}}, W_{\text{eff}}$ and two coupling functions A, B, all of the depending on the dynamical scalar fields ϕ, R (in turn, related to the original three scalars, the dilaton and 2 shapes). Writing down the evolution equations for gravity and both scalars, and going to the cosmological setting via standard steps one gets the following set of equations.

The so called Friedmann equation for Hubble parameter $H = \dot{a}(t)/a(t)$ where $a(t)$ is the Universe scale factor reads

$$H^2 = \frac{8\pi G}{3}(\rho_1 + \rho_2 + V_{\text{eff}} + W_{\text{eff}}) \quad (14)$$

$$+ \frac{2\alpha_d^2}{1 + 2\alpha_d^2}\dot{\phi}^2 + \frac{1}{1 + 2\alpha_d^2}\dot{R}^2. \quad (15)$$

The second Einstein equation is

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_1 + 3p_1 + \rho_2 + 3p_2 - 2V_{\text{eff}} - 2W_{\text{eff}})$$

$$-\frac{4\alpha_d^2}{1 + 2\alpha_d^2}\dot{\phi}^2 - \frac{2}{1 + 2\alpha_d^2}\dot{R}^2 \quad (16)$$

The field equations for R and ϕ read

$$\ddot{R} + 3H\dot{R} = -8\pi G \frac{1 + 2\alpha_d^2}{6}[\frac{\partial V_{\text{eff}}}{\partial R} + \frac{\partial W_{\text{eff}}}{\partial R}$$

$$+\alpha_R^{(1)}(\rho_1 - 3P_1) + \alpha_R^{(2)}(\rho_2 - 3P_2)] \quad (17)$$

$$\ddot{\phi} + 3H\dot{\phi} = -8\pi G \frac{1 + 2\alpha_d^2}{12\alpha_d^2}[\frac{\partial V_{\text{eff}}}{\partial \phi} + \frac{\partial W_{\text{eff}}}{\partial \phi}$$

$$+\alpha_\phi^{(1)}(\rho_1 - 3P_1) + \alpha_\phi^{(2)}(\rho_2 - 3P_2)]. \quad (18)$$

In these expressions the matter on both branes has energy densities and pressures ρ_i, P_i entering for scalars is the trace of the energy–momentum tensor for each brane's matter.

The coupling functions $\alpha_\phi^{(i)}$ and $\alpha_R^{(i)}$ are defined as

$$\alpha_\phi^{(1)} = \frac{\partial \ln A}{\partial \phi}, \quad \alpha_\phi^{(2)} = \frac{\partial \ln B}{\partial \phi}; \quad (19)$$

$$\alpha_R^{(1)} = \frac{\partial \ln A}{\partial R}, \quad \alpha_R^{(2)} = \frac{\partial \ln B}{\partial R}. \quad (20)$$

and in the model with exponential potential they have a particular form, depending on the parameter α_d of the model. These functions describe mutual attraction of the two branes, and thus the distance between them.

In Fig.3(a) one finds cosmic evolution of matter composition of this model, which is tuned to reproduce durations of radiation, matter and scalar-dominated eras. Fig.3(b) shows the corresponding motion of the two branes. As one should expect from discussion in the preceding section, one finds some sliding behavior, from equilibrium at radiation dominated era to a gentle slide during matter era to another change recently (related to scalar dominance and accelerated Universe, as now observed.

The main lesson from this is that the two-brane model leads to nearly the same cosmological eqns as people were discussing on more general grounds for ad hoc scalars. Therefore, the ideas about cosmological attractor leading to self decoupling of the scalars work. In particular, in radiation-dominated era there is no "force" term in the r.h.s. as $\rho = 3P$ and rolling to the minimum kills evolution fast. Matter dominated period start changes in the scalars again, as well as recent transition from matter to cosmological constant dominance.

An ideal model would (i) produce the potential at the current minimum equal to the observed cosmological constant; (ii) make relaxation to it so good that all current and recent (Oklo, meteorites) strong limits on any modifications of the constants be naturally satisfied; and maybe (iii) produce interesting modifications at redshift $z \sim 1$ where the matter composition at our brane had changed the last time.

A particular potentials studied in [10] numerically have not achieved these goals (and in fact the authors found some unexpected theoretical problems on the way). But the game is too recent to give up on it, and the settings itself is clearly sufficiently flexible to try many variants.

IV. CHANGING PHYSICS NEAR MASSIVE BODIES

The reason gravity is so important at large scales is that its effect is additive. The same should be true for massless (or very light) scalars: its effect near large body is proportional to the number of particles in it.

Unlike in the brane models, in which one can only hope to feel motion of the second brane via constant variations, we do see the Sun. But suppose we cannot (e.g. there are always clouds) and the temperature is also unchanged. Can one still feel that the Sun gets closer or further, in a periodic fashion? By an oscillating physical constants, perhaps.

A. Scalars coupled to gravity in static case

For static star (or Galaxy) the eqn for the dilaton is

$$\Delta\phi = 4\pi G A'(\phi) T \qquad (21)$$

to be of course complimented by Einstein's eqn to which it is coupled

$$R_{\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi + 8\pi G(T_{\mu\nu} - (T/2)g_{\mu\nu}) \qquad (22)$$

It is a nonlinear eqn inside the body, which as usual for stars should be solved starting from r=0 outward. But what one should expect to find for dilaton outside the body?

If there is no matter there, $T = 0$ at $r > R$, then the only possibility is simply a combination of a Coulomb-like field and constant

$$\phi = Q_d/r + \phi_0 \qquad (23)$$

Constant part is the cosmological dilaton expectation value we discussed above: the Coulomb-like field 1/r is the subject of this section.

But before we discuss it, let us think what would happen at constant density, say if one wish to include some matter such as interstellar gas or cosmological constant, which produces small but nonzero T everywhere? If one writes $A'(\phi) = \beta\phi$ the r.h.s. is then a mass term with the dilaton mass

$$m_d^2 = 4\pi G \beta T \qquad (24)$$

If T is given by the cosmological constant this mass becomes $m_d^2 = 6\beta\Omega_c H^2$ where $\Omega_c = 0.7$ is its current balance in total density and H is current Hubble constant. Ignoring factors of the order one one finds that this mass is only important for sizes as large as the visible Universe itself. Thus, the first lesson thing we learned from this is that the sign of β should be such that there is no instability present in the Universe: we have to be near the minimum, not maximum of the potential.

The other lesson is that for any body the effective dilaton mass can only gets important if its size is close to its own relativistic radius. It is indeed precisely in this case, for which Damour and Esposito-Farese [22] argued that large dilaton-induced effect is possible inside neutron stars, provided the coupling β happen to be specially tuned to the star parameters making it into a kind or resonator amplifying small cosmological dilaton field outside.

In fact we know for sure that *outside* neutron stars the dilaton field is small. It was shown in multiple works (see refs e.g. in [22]) that the absence of scalar-induced corrections for binary pulsar motion can get significant limits on that, although they are still somewhat weaker than the limit $A' < 10^{-3}$ following from solar system experiments related to post-Newtonian gravity effects.

However impressive is the accuracy of the available data on motion of some binary pulsars, we think that

potentially spectroscopic studies of scalar-induced modification of physics constants is a more promising way to go for future studies, as accuracy of spectral line frequencies is still much higher.

For not-too-relativistic objects, like the usual stars or planets, both their total mass M and the total dilaton charge Q are simply proportional to the number of nucleons in them, and thus the dilaton field is simply proportional to the gravitational potential

$$\phi - \phi_0 = \kappa(GM/rc^2) \qquad (25)$$

and thus we expect that the fundamental constants would also depend on the position via the gravitational potential at the the measurement point.

B. Comparison between potential and real experiments

Naively, one may think that the larger is the dimensionless gravity potential (GM/rc^2) of the object considered, the better. However, different objects allow for quite different accuracy.

Let us mention few possibilities, using as a comparison parameter the product of gravity potential divided by the tentative relative accuracy

$$P = (GM/rc^2)/(accuracy) \qquad (26)$$

(i) Gravity potential on Earth is changing due to ellipticity of its orbit: the corresponding variation $\delta(GM/rc^2) = 3.3 * 10^{-10}$. The accuracy of atomic clocks in laboratory conditions approaches 10^{-16}, and so $P \sim 3 * 10^6$. However, comparing a clock on Earth and distant satellite one may get $\delta(GM/rc^2) \sim 10^{-9}$ and $P \sim 10^7$. The space mission was recently discussed, e.g. in the proposal [18] and references therein.

(ii) Sun (or other ordinary stars) has $GM/rc^2 \sim 2 * 10^{-7}$. Assuming accuracy 10^{-7} in the measurements of atomic spectra near the surface we get $P \sim 1$. However, a mission with modern atomic clocks sent to the Sun would have $P \sim 10^8$ or so, see details in the proposal [17].

(iii) The stars at different positions inside our (or other) Galaxy have gravitational potential difference of the order of 10^{-7}, and (like for the Sun edge) one would expect $P \sim 1$. Clouds which give the observable absorption lines in quasar spectra have also different gravitational potentials (relative to Earth), of comparable magnitude.

(iv) White/brown dwarfs have $GM/rc^2 \sim 3 * 10^{-4}$, and in some cases rather low temperature. We thus get $P \sim 3 * 10^3$.

(v) Neutron stars have very large gravitational potential $GM/rc^2 \sim .1$, but high temperature and magnetic fields make accuracy of atomic spectroscopy rather problematic, we give tentative accuracy 1 percent. $P \sim 10$.

(vi) Black holes, in spite of its large gravitational potential, have no scalar field outside the Shwartzschield radius, and thus are not useful for our purpose.

Accuracy of the atomic clocks is so high because they use extremely narrow lines. At this stage, therefore, star spectroscopy seem not to be competitive: the situation may change if narrow lines be identified.

Now let us see what is the best limit available today. As an example we consider recent work [20] who obtained the following value for the half-year variation of the frequency ratio of two atomic clocks: (i) optical transitions in mercury ions $^{199}Hg^+$ and (ii) hyperfine splitting in ^{133}Cs (the frequency standard). The limit obtained is

$$\delta ln(\frac{\omega_{Hg}}{\omega_{Cs}}) = (0.7 \pm 1.2) * 10^{-15} \qquad (27)$$

For Cs/Hg frequency ratio of these clocks the dependence on the fundamental constants was evaluated in [21] with the result

$$\delta ln(\frac{\omega_{Hg}}{\omega_{Cs}}) = -6\frac{\delta\alpha}{\alpha} + 0.04\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} - \frac{\delta(m_e/\Lambda_{QCD})}{(m_e/\Lambda_{QCD})} \qquad (28)$$

Another work [23] compare H and ^{133}Cs hyperfine transitions. The amplitude of the half-year variation found were

$$\delta ln(\omega_H/\omega_{Cs}) < 7 * 10^{-15} \qquad (29)$$

The sensitivity [21]

$$\delta ln(\frac{\omega_H}{\omega_{Cs}}) = 0.83\frac{\delta\alpha}{\alpha} + 0.11\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} \qquad (30)$$

There is no sensitivity to m_e/Λ_{QCD} because they are both hyperfine transitions.

As motivated above, we assume that scalar and gravitational potentials are proportional to each other, and thus introduce parameters k_i as follows

$$\frac{\delta\alpha}{\alpha} = k_\alpha \delta(\frac{GM}{rc^2}) \qquad (31)$$

$$\frac{\delta(m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = k_q \delta(\frac{GM}{rc^2}) \qquad (32)$$

$$\frac{\delta(m_e/\Lambda_{QCD})}{(m_e/\Lambda_{QCD})} = \frac{\delta(m_e/m_p)}{(m_e/m_p)} = k_e \delta(\frac{GM}{rc^2}) \qquad (33)$$

where in the r.h.s. stands half-year variation of Sun's gravitational potential on Earth.

In such terms, the results of both experiments can be rewritten as

$$k_\alpha + 0.17k_e = (-3.5 \pm 6) * 10^{-7} \qquad (34)$$

$$|k_\alpha + 0.13k_q| < 2.5 * 10^{-5} \qquad (35)$$

The sensitivity coefficients for other optical clocks can be found in Refs. [24, 25], for the hyperfine clocks in

Ref. [21]. The sensitivity coefficients may be very large in transitions between very close levels in Dysprosium atom [24, 26] ($\delta ln(\omega_{Dy}/\omega_{Cs}) \sim 10^8 \delta\alpha/\alpha$, see similar cases in [27]), molecules ($\sim 10-1000$) [28], and optical UV transi-

tion in ^{229}Th nucleus ($\sim 10^5$). The sensitivity coefficients may also be very large in collision of cold molecules near Feshbach resonance ($\sim 10^2 - 10^{12}$) [30].

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[31] Note the similarity with expanding the view from the 2-d Earth surface, our natural habitat, to 4d Einsteinian Universe. The difference is: we can see stars and galaxies, but we dont know anything definite about these extra dimensions yet.

[32] There are also some other stability conditions we dont mention, see [6, 13].

[33] our brane one may call "the brane Earth" and the other "the brane Sun", to enhance analogy to Copernicus/Kepler system of the world. It also explains which one has the dominant mass.

[34] Note that simple additivity of contributions to energy-momentum tensor is ambiguous for non-gravitationally interacting species, but we dont expect it to be a problem for the case we aimed at, in which one component is baryonic/dark matter and the other the dark energy.

[35] A prime in $A'(\phi)$ is still a derivative over ϕ though: we hope it will not confuse the reader.